Localization Capabilities of Next Generation Gravitational Wave Detectors

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Abstract

Localization of gravitational wave sources is extremely important for any type of multimessenger followup. Recent studies have shown that in the era of next-generation detectors, a handful of such sources can be localized even using only one detector. This is possible due to modulations of the signal due to the long arms of the detector and the apparent motion of the source due to the rotation of the Earth. However, those studies include only the dominant quadrupolar mode while such signals might have significant contributions from higher modes. Higher modes with an azimuthal number greater than 2 also last longer in the band making the rotation of Earth even more important. We study the role of these higher modes in localization.

1. Introduction

The LIGO-Virgo-Kagra (LVK) (Aasi et al. 2015; Acernese et al. 2014; Akutsu et al. 2020) collaboration has confidently detected gravitational waves (GW) from nearly 100 compact binary coalescences (CBC), including binary neutron stars (BNS) (Abbott & et al. 2017; Abbott et al. 2020), binary black holes (BBH) (Abbott et al. 2021c; Nitz et al. 2021) and neutron star black holes (NSBH) (Abbott et al. 2021a). GW170817, a BNS event also had electromagnetic counterparts ranging from gamma-ray to optical (Abbott et al. 2017a,b) enabling us to probe several fundamental theories about the Universe (Abbott et al. 2021b,d,e).

Electromagnetic follow-up of a gravitational wave source is only possible when the source has been localized on the sky. A single current-generation gravitational wave detector cannot localize a source as all the geometric parameters of the source like right ascension, declination, polarisation angle, inclination, and luminosity distance are degenerate in the amplitude of a GW waveform. Multiple detectors are required to triangulate the source, breaking the degeneracy between the extrinsic parameters.

A well-localized GW source also has precise measurements of luminosity distance. Unlike the electromagnetic (EM) measurement of distance, this measurement is free from calibration uncertainties existing in the cosmic distance ladder. Such a measurement shall be crucial for precision cosmology and test our understanding of calibration uncertainties in the cosmic distance ladder.

Future detectors like the Cosmic Explorer (CE) (Reitze et al. 2019) and the Einstein Telescope (ET) (Punturo & et al 2010) shall be able to localize loud sources even with one detector. These detectors have longer arms and much better response at lower frequencies. Sensitivity at lower frequency means that gravitational wave signals from BNS might lie in-band for about an hour making the source move across the sky due to Earth's rotation. Long arms compel us to calculate the travel time of a GW across the arms of the detector beyond the static limit, where the wavelength of a gravitational wave is assumed to be infinity (Rakhmanov et al. 2008; Rakhmanov 2009). These effects make the antenna response frequency dependent, which breaks the degeneracy between extrinsic parameters, enabling us to localize sources in the sky using only one detector.

Recently we performed a Bayesian parameter estimation (PE) using `Bilby`, a commonly used PE pipeline, on simulated signals with an optimal signal-to-noise ratio (SNR) of 1000 (Baral et al. 2023). To study the localization capabilities of BNS mergers using a single CE, detector modes other than the dominant (2,2) mode were neglected in this work. However, they might have finite contributions given the sensitivity of the future detectors. The length of the signal makes likelihood evaluations computationally costly. A multibanding technique (Morisaki 2021), a form of adaptive sampling of the waveform, has been used in this work giving a speedup by a factor of 500 compared to traditional methods. This makes PE with all relevant effects in CE feasible.

Due to the increased sensitivity of the next-generation GW detectors, higher-order multipoles (HM) of emission in addition to the dominant quadruple (2,2) mode are expected to be detected. During the inspiral phase the frequency of a mode (l, m) is given by $m\Omega$, where Ω is the orbital frequency. The frequency of each of these multipole m corresponds to a (2,2) frequency of $2f/m$ (London et al. 2018). This means that the time to merger of a pure (3,3) mode from 6 Hz is equal to the time to merger of a pure (2,2) mode from 4 Hz. Thus modes with higher multipole last longer in-band and make effects due to the rotation of Earth more pronounced which shall imply better sky localization. In this work, we modify the codes used by Baral et al. (2023) to study the effects of higher modes in source localization for the next generation groundbased GW observatories, including effects due to detector size and the rotation of the Earth. The modifications are presented in detail in the following section.

2. Methods

2.1. Generation of the simulated dataset We perform zero-noise injections in the projected CE power spectral density (PSD) using IMRPhenomHM. Each mode has been computed separately in the frequency domain with the antenna pattern computed individually for each mode using methods outlined in Baral et al. (2023). Unlike Baral et al. (2023) the tidal parameters are ignored due to the lack of waveforms containing higher modes and tidal parameters. However, we do not expect tidal parameters to have much effect on sky localization posteriors and so can be ignored safely for a first work of this kind.

2.2. The likelihood function Two modifications are required to the likelihood function presented in Baral et al. (2023). The first one is to calculate the waveform for every mode and then sum it up as described in the previous section. However, to speed things up we group modes containing the same azimuthal mode number (m). This does not create issues in our analysis as the antenna pattern only depends on m.

Baral et al. (2023) uses the analytic phase marginalizer implemented in `BILBY` for phase marginalization. However, this is only accurate if the waveform only contains the (2,2) mode. So we need to come up with a new algorithm for phase marginalization for this work. The details are described in the following section.

2.3. Phase marginalization for 22+33 mode We try to come up with a scheme to perform phase marginalization for a waveform containing $(2, 2)$ and $(3, 3)$ modes. The gravitational wave is given by,

$$\mu \equiv \mu_2 + \mu_3 \tag{1}$$
$$\mu_m \equiv \mu_{mm} + \mu_{m-m} = e^{im\phi_c}\mu(\phi_c = 0) \tag{2}$$
$$\mu_{lm} \equiv F_+\mathrm{Re}\big(h_{lm}\ {}_{-2}Y_{lm}\big) + F_\times\mathrm{Im}\big(h_{lm}\ {}_{-2}Y_{lm}\big) \tag{3}$$

The likelihood is defined by,

$$\begin{aligned}
\mathcal{L}^{\phi_c}_{\mathrm{marg}} &= \int_0^{2\pi} d\phi_c \exp\Big(\frac{1}{2}\langle d, \mu\rangle + \frac{1}{2}\langle \mu, d\rangle\Big)\pi(\phi_c) + ... & (4)\\
&= \int_0^{2\pi} d\phi_c \exp\Big(\frac{1}{2}\langle d, \mu_2(\phi_c = 0)\rangle e^{2i\phi_c} + \frac{1}{2}\langle \mu_2, d\rangle e^{-2i\phi_c} + \frac{1}{2}\langle d, \mu_3(\phi_c = 0)\rangle e^{-3i\phi_c} + \frac{1}{2}\langle \mu_3, d\rangle e^{3i\phi_c}\Big)\pi(\phi_c) + ... & (5)\\
&= \int_0^{2\pi} \frac{d\phi_c}{2\pi} \exp\big(A_2\cos(2\phi_c) + B_2\sin(2\phi_c) + A_3\cos(3\phi_c) + B_3\sin(3\phi_c)\big)\pi(\phi_c) + ... & (6)\\
&= \int_0^{2\pi} \frac{d\phi_c}{2\pi} \exp\big(f_2 + f_3\big)\pi(\phi_c) + ... & (7)
\end{aligned}$$

where

$$\begin{aligned}
A_m &\equiv Re\langle d, \mu_m(\phi_c = 0)\rangle & (8)\\
B_m &\equiv Im\langle d, \mu_m(\phi_c = 0)\rangle & (9)\\
f_m &\equiv A_m\cos(m\phi_c) + B_2\sin(m\phi_c) & (10)
\end{aligned}$$

The most trivial solution is to integrate using a trapezoidal integrator with a finely placed grid. The integration is relatively computationally cheap compared to waveform evaluation. Moreover, $f \equiv f_1 + f_2$ is sharply peaked around $\phi_c = p_1$ and $\phi_c = p_2$. Therefore the integral might be evaluated by saddle point approximation (SPA) which is given by,

$$\mathcal{L}_{\rm marg}^{\phi_c} = \frac{1}{2\pi}\left[e^{f(p_1)}\sqrt{\frac{2\pi}{-f''(p_1)}} + e^{f(p_2)}\sqrt{\frac{2\pi}{-f''(p_2)}}\right] + ... \tag{11}$$

Since $f(p_1)$ and $f(p_2)$ are an order of $1e6$ above expression will result in overflow errors. However, we need the log of the integrand which can be evaluated as,

$$f(p_1) + \log\left[\sqrt{\frac{2\pi}{-f''(p_1)}} + e^{f(p_2)-f(p_1)}\sqrt{\frac{2\pi}{-f''(p_2)}}\right]$$

2.4. Generalization to other modes In this section, we try to generalize the treatment in the previous section to include all modes allowed by the waveform class. The gravitational wave is given by,

$$\mu \equiv \mu_2 + \mu_3 \tag{12}$$
$$\mu_m \equiv \mu_{mm} + \mu_{m\ -m} + \mu_{m+1\ m} + \mu_{m+1\ -m} \tag{13}$$
$$= e^{im\phi_c}\mu(\phi_c = 0) \tag{14}$$
$$\mu_{lm} \equiv F_+ \mathrm{Re}\big(h_{lm\ -2}Y_{lm}\big) + F_\times \mathrm{Im}\big(h_{lm\ -2}Y_{lm}\big) \tag{15}$$

The likelihood is defined by,

$$\mathcal{L}_{\rm marg}^{\phi_c} = \int_0^{2\pi} d\phi_c \exp\big(\frac{1}{2}\langle d,\mu\rangle + \frac{1}{2}\langle \mu, d\rangle\big)\pi(\phi_c) + ... \tag{16}$$
$$= \int_0^{2\pi} d\phi_c \exp\big(\sum_m \frac{1}{2}\langle d,\mu_m(\phi_c=0)\rangle e^{im\phi_c} + \frac{1}{2}\langle \mu_m(\phi_c=0), d\rangle e^{-im\phi_c}\big)\pi(\phi_c) + ... \tag{17}$$
$$= \int_0^{2\pi} \frac{d\phi_c}{2\pi} \exp\big(\sum_m A_m\cos(m\phi_c) + B_m \sin(m\phi_c)\big) + ... \tag{18}$$

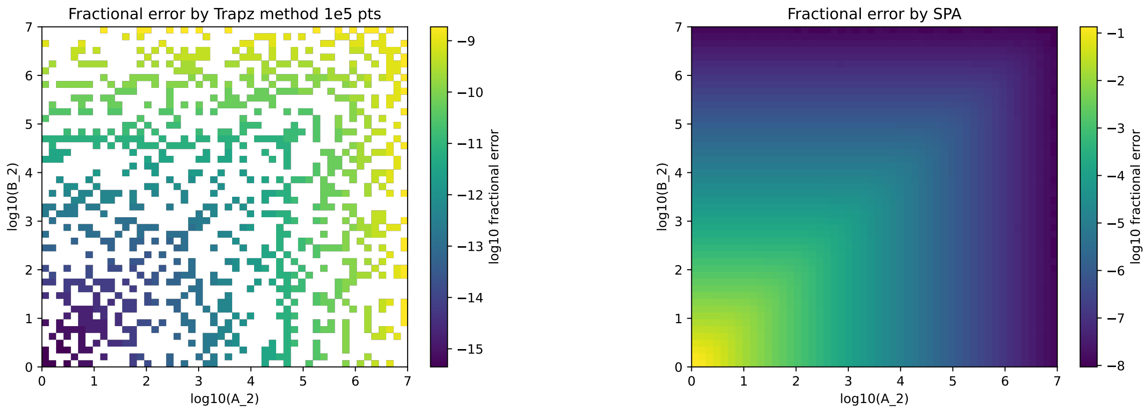
where

$$A_m \equiv Re\langle d, \mu_m(\phi_c = 0)\rangle \tag{19}$$
$$B_m \equiv Im\langle d, \mu_m(\phi_c = 0)\rangle \tag{20}$$

3. Results

3.1. Choosing an appropriate integrator for phase marginalization We perform the phase marginalization integration (equation 18) using trapezoidal integration using 1e5 points and using SPA. We only take the 2,2 mode (which is equivalent to setting all A_ms and B_ms to zero if m≠2) as that is the dominant mode and the integrand can be performed analytically. We plot the fractional errors from the true analytic values in figure 1. For an SNR of 1000, we do not expect A_m or B_m to be over $1e7$. In this range, the error due to trapezoidal integrator is always less than $1e-9$ while for SPA it varies between 0.1 to $1e$-8. Clearly the trapezoidal integrator works better. The time taken due to integration is not the dominant cost and hence does not matter.

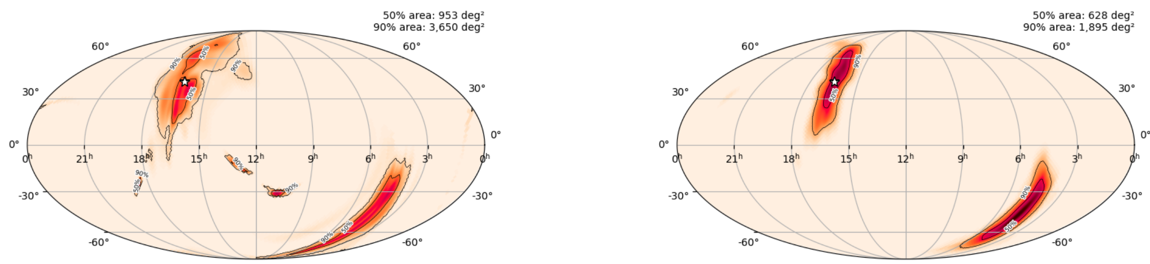
3.2. Posteriors and skymaps We perform parameter estimation on an asymmetric neutron star system (Primary mass: $2M_\odot$; Secondary mass: $1M_\odot$) using the dominant 2,2 mode (Figure 3) and the 2,2 + 3,3 mode (Figure 4). Asymmetric masses are chosen to increase contributions from higher modes. Addition of the the 3,3 mode leads to much improved sky localization as seen in Figure 2. In general, adding another mode makes parameter recovery better as can be seen from the posteriors.



(a) Trapezoidal Integration

(b) Saddle Point Approximation

Figure 1: Fractional errors due to trapezoidal integration (left) and SPA (right). As expected trapezoidal is more accurate for lower values of A_m and B_m while the trend is reversed for SPA. The white places in the left plot are because the numerical trapezoidal integrator and the analytic integrator return the exact same value upto the default float precision of Python.



(a) Skymap obtained by using only the 2,2 mode

(b) Skymap obtained by using the 2,2 + 3,3 mode

Figure 2: Skymap obtained by full Bayesian PE runs. Multimodality exists in the parameter space, due to the fact we are using only one detector.

4. Discussion

We perform parameter estimation taking into account the 3,3 mode and obtain improved parameter recovery, better localization, and unbiased skymaps. For some sources, specifically the ones with symmetric masses the 4,4 mode becomes the dominant mode after the 2,2 mode. Presently I am working on such sources. The main challenge of incorporating the 4,4 mode is the computational cost as the waveforms become extremely long. Methods such as relative binning may become essential to perform such PE runs in a reasonable amount of time.

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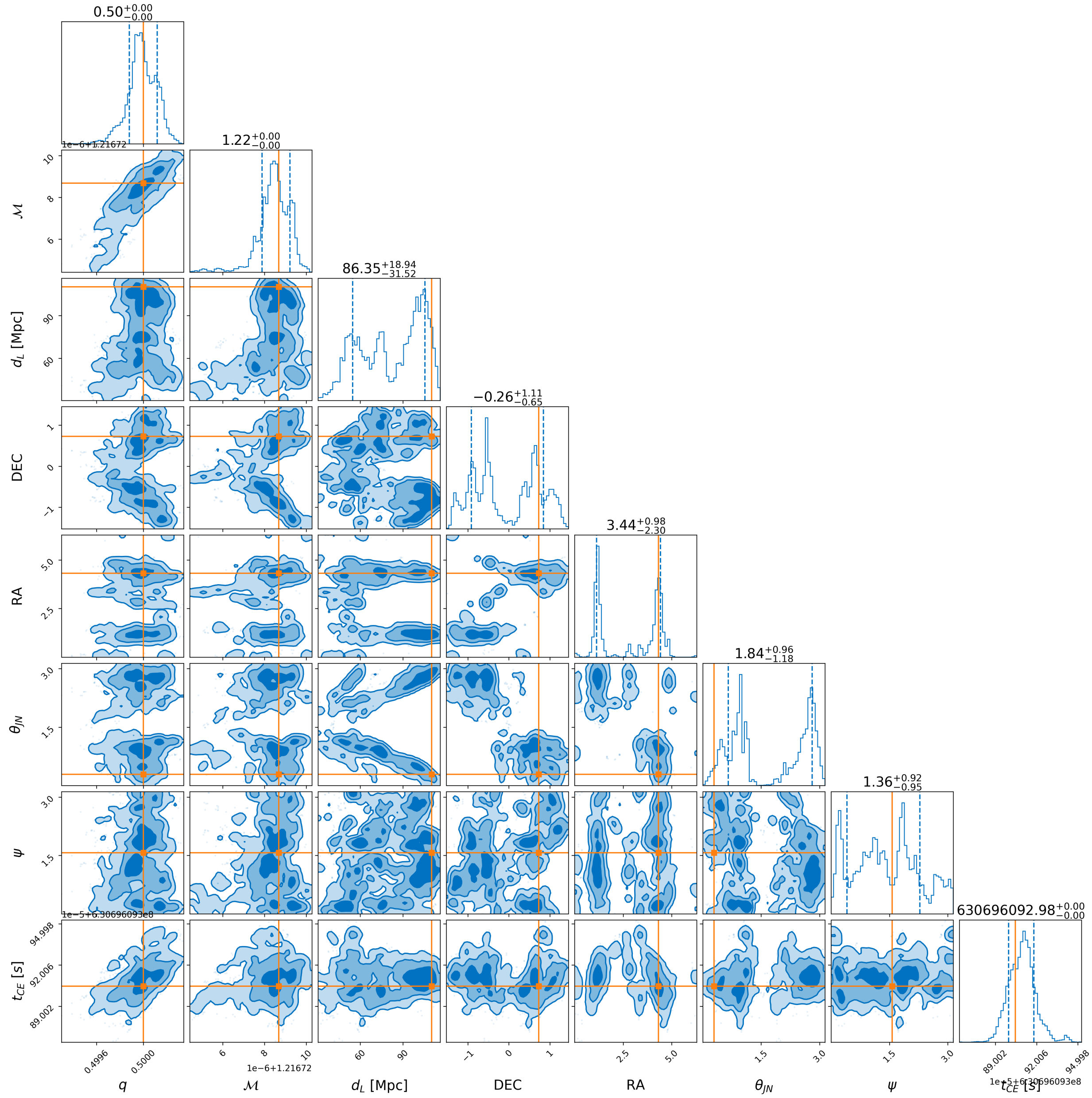

Figure 3: Corner plot for the PE run that takes only the 2,2 mode into account.

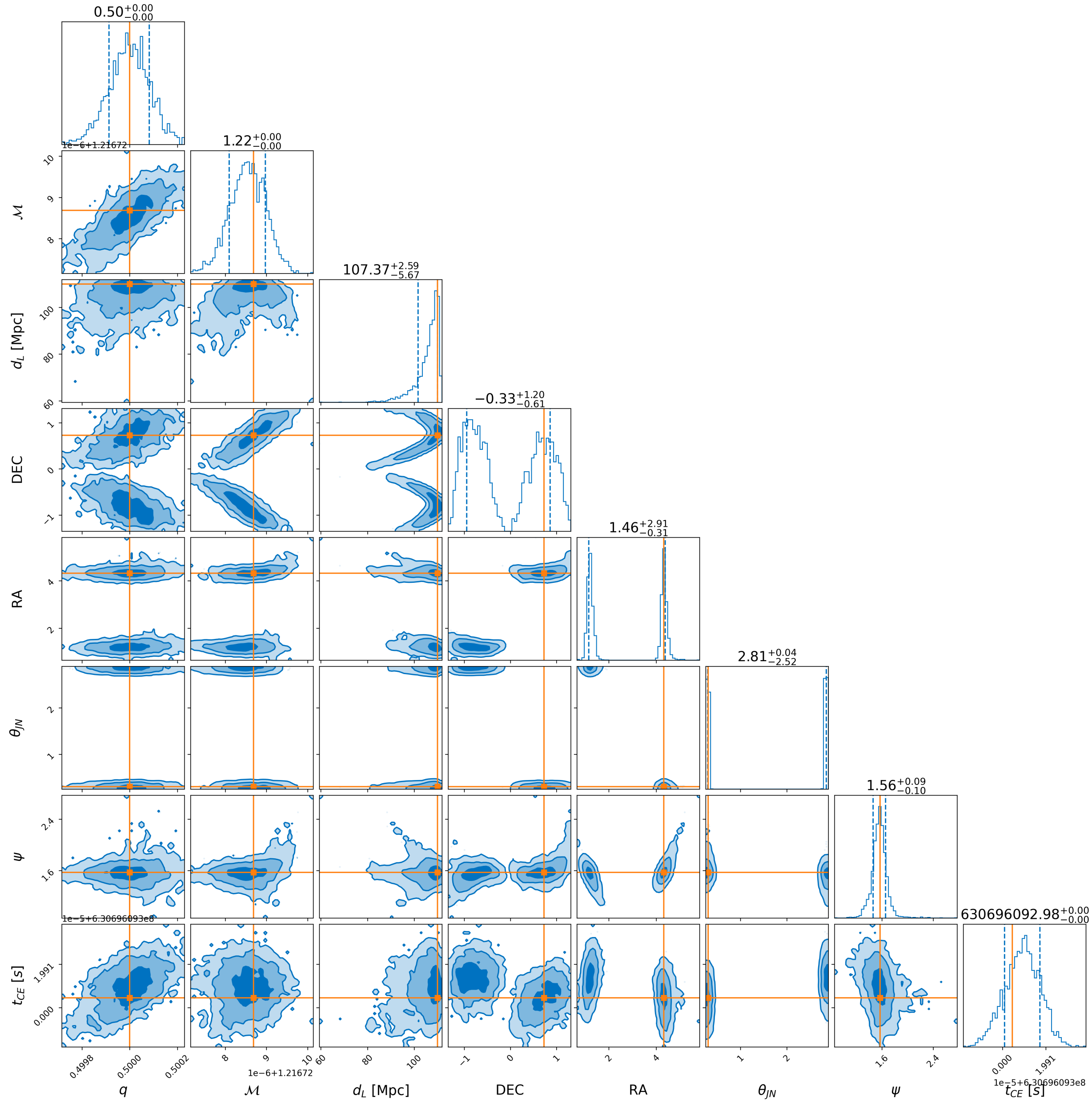

Figure 4: Corner plot for the PE run that takes the 2,2 mode and the 3,3 mode into account.

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